

## The optimum seeking method of Pavement maintenance based on interval fuzzy soft sets

Wu Congliang<sup>1</sup>, Yang Xiaoye<sup>2</sup>

School of Traffic and transportation<sup>1</sup>, School of civil engineering & architecture<sup>2</sup>, Chongqing Jiaotong University, Chongqing, China

### Abstract

For pressed for time or limited funding, Policy-makers may not know the accurate value of evaluation indexes, and only know the interval-number of the evaluation index, in determining the order of pavement maintenance. Aiming at this situation, the decision model, making decisions based on the scope of the evaluation index values, of pavement maintenance is established. Model adopts the analytic hierarchy process (AHP) to determine the weight of evaluation index, and sorts through the decision-making method of interval fuzzy soft sets. At last, the same sort result with other methods was obtained by an example to prove the feasibility of the model.

**Key Words:** pavement maintenance, interval-number, soft sets, decision model

### I. INTRODUCTION

The road performance will gradually deteriorate as it bears the effect of the traffic loads and environmental factors, so it is important to maintenance and repair the road timely and effective. In the case of restricted budgets, the priorities of road maintenance should be considered. Sorting, widely used, is one of the most important methods in the pavement management system around the world<sup>[1]</sup>. But, at present, the majority of sorting method is based on the situation of the evaluation index attribute value is known, does not take into account the case of only knowing the interval-number of the evaluation index<sup>[2]</sup>. And, sometimes it is very difficult to know the exact value of the index attribute. Meanwhile, time is short, it is not necessary to investigate all roads within the road network in detail. So, to select the most in need of maintenance project by knowing the interval-number of the evaluation index, which can achieve the result that to save time, energy and money.

### II. To establish the interval-number decision model

Assuming that the number of pavement need maintenance or reconstruction is  $m$ , the alternative project sets  $h = \{ h_1, h_2, \dots, h_i \}$ ,  $i \in m$ ,  $m = \{ 1, 2, \dots, m \}$ ,  $m \geq 2$ ; there are  $n$  indicators reflecting pavement performance, so the decision attribute set  $\varepsilon = \{ \varepsilon_1, \varepsilon_2, \dots, \varepsilon_j \}$ ,  $j \in n$ ,  $n = \{ 1, 2, \dots, n \}$ ,  $n \geq 2$ ; corresponding index  $\varepsilon_j$ , the attribute value of project  $h_i$  is interval-number  $[a_{ij}^-, a_{ij}^+]$ ,  $a_{ij}^-$  is lower limit value,  $a_{ij}^+$  is ceiling value, in which  $i=1,2,\dots,m$ ;  $j=1,2,\dots,n$ . The decision-making information matrix constructed by original data is  $A = (a_{ij})_{m \times n}$ ,  $a_{ij} = [a_{ij}^-, a_{ij}^+]$ .

#### 2.1 decision matrix standardized treatment

Efficiency type attribute: pavement condition index PCI, lateral force coefficient SFC, etc; cost attribute: roughness index IRI, pavement crack rate, etc. The model utilize the Interval-numbers decision matrix standardization method in literature<sup>[3]</sup> to standardize the two kinds of attribute:

Efficiency attribute index was calculated by the following equation:

$$b_{ij}^- = \frac{a_{ij}^- - \min_{1 < i < m} (a_{ij}^-)}{\max_{1 < i < m} (a_{ij}^+) - \min_{1 < i < m} (a_{ij}^-)}$$

$$b_{ij}^+ = \frac{a_{ij}^+ - \min_{1 < i < m} (a_{ij}^+)}{\max_{1 < i < m} (a_{ij}^+) - \min_{1 < i < m} (a_{ij}^-)} \dots \dots \dots \text{equation (1)}$$

Cost type attribute index according to the following equation:

$$b_{ij}^- = \frac{\max_{1 < i < m} (a_{ij}^+) - a_{ij}^+}{\max_{1 < i < m} (a_{ij}^+) - \min_{1 < i < m} (a_{ij}^-)}$$

$$b_{ij}^+ = \frac{\max_{1 < i < m} (a_{ij}^+) - a_{ij}^-}{\max_{1 < i < m} (a_{ij}^+) - \min_{1 < i < m} (a_{ij}^-)} \dots \dots \dots \text{equation (2)}$$

$j=1,2,\dots,n_0$

After normalization of attributes, decision-making information matrix composed of the original data  $A=(a_{ij})_{m \times n}$  is converted into standardized matrix  $B=(b_{ij})_{m \times n}$ ,  $b_{ij} = [b_{ij}^-, b_{ij}^+]$ .

## 2.2 To determine decision attribute weights<sup>[4]</sup>

Determine the attribute weights Through the analytic hierarchy process (AHP) is  $(w_1, w_2, \dots, w_n)$ , and  $\sum_{j=1}^n w_j = 1, w_j \geq 0$ , so the interval Numbers decision evaluation matrix is  $R = (r_{ij})_{m \times n}$ ,  $r_{ij} = [r_{ij}^-, r_{ij}^+]$ .

$$R = (b_{ij}) * w_j = [b_{ij}^- * w_j, b_{ij}^+ * w_j]$$

$$= \begin{bmatrix} [b_{11}] * w_1 & [b_{12}] * w_2 & \dots & [b_{1n}] * w_n \\ [b_{21}] * w_1 & [b_{22}] * w_2 & \dots & [b_{2n}] * w_n \\ \dots & \dots & \dots & \dots \\ [b_{m1}] * w_1 & [b_{m2}] * w_2 & \dots & [b_{mn}] * w_n \end{bmatrix}$$

## 2.3 The fuzzy soft set decision method

### 2.3.1 The theory

Definition 1<sup>[5]</sup> U is the initial field, E is the parameter set. Sequence of (F, E) is called soft set if and only if F is a mapping of power set from E to set U, namely  $F: E \rightarrow P(U)$ ,  $P(U)$  is the power set of U.

Definition 2<sup>[6]</sup> U is the initial field, E is a set of parameters,  $\xi(U)$  said all of fuzzy subset collection on U. Make  $A \in E$ , the sequence of (F, A) is known as a basic fuzzy soft set on U, F is a mapping,  $F: A \rightarrow \xi(U)$ .

In short, a fuzzy soft set is Parameter set composed of fuzzy subsets on field U. if  $\varepsilon \in A$ .  $F(\varepsilon)$  can be regarded as A fuzzy soft set of the  $\varepsilon$

approximation of fuzzy set (F, A).

### 2.3.2 Decision-making method

Firstly, according to the decision method in literature<sup>[7]</sup>, to build basic fuzzy soft set (F, E). As table 1:

**Table1.** The tabular form of basic fuzzy Soft Set (F,E)

U	$\varepsilon_1$	$\varepsilon_2$	...	$\varepsilon_n$
$h_1$	$[r_{11}^-, r_{11}^+]$	$[r_{12}^-, r_{12}^+]$	...	$[r_{1n}^-, r_{1n}^+]$
$h_2$	$[r_{21}^-, r_{21}^+]$	$[r_{22}^-, r_{22}^+]$	...	$[r_{2n}^-, r_{2n}^+]$
...	...	...	...	...
$h_m$	$[r_{m1}^-, r_{m1}^+]$	$[r_{m2}^-, r_{m2}^+]$	...	$[r_{mn}^-, r_{mn}^+]$

Table  $\{h_1, h_2, \dots, h_i\}$  is field, namely all alternatives of the multiple attribute decision making problems;  $\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_j\}$  as the parameter set, namely all decision attribute of the multiple attribute decision making problems;  $r_{ij}$  said attribute value of decision-making objects (alternatives)  $h_i$  about the parameter (decision attribute)  $\varepsilon_j$ .

Secondly, according to the data in table 4, calculate choice value  $C_i$  of decision-making object  $h_i$  of the basic fuzzy soft set (F, E). Option value calculated by the next equation:

$$C_i = [u_i, v_i] = [\sum_{j=1}^m r_{ij}^-, \sum_{j=1}^m r_{ij}^+] \dots \dots \dots \text{Equation(3)}$$

Where  $r_{ij}$  means fuzzy comprehensive evaluation value of object  $h_i$  about parameter  $\varepsilon_j$ , m said the number of parameters.

Finally, according to the choice value of all decision-making object, calculated decision values  $r_i$  of decision object (alternatives)  $h_i$  ( $\forall h_i \in U$ ). Decision value  $r_i$  were calculated by the next equation:

$$r_i = \sum_{h_j \in U} ((u_i - u_j) + (v_i - v_j)) \dots \dots \dots \text{Equation(4)}$$

so the object have maximum decision value  $r_i$  should be maintained at the earliest.

## III. example

The feasibility and effectiveness of basic fuzzy soft set multiple attribute decision making method be

verified by the example of road maintenance decision problem in the paper [8].

The survey data of asphalt pavement using state in a region as table 2, sorting the 5 need maintenance road.

Tab.2 The data of road states

road	SFC	Crack rate/%	IRI
1	25.8	3.7	2.85
2	20-25	0.5-2	3-4
3	25-30	1-3	3-5
4	18-22	1-3	4-6
5	20-25	3-5	2-4

Decision-making information matrix A can be obtained through the table 2:

$$A = \begin{bmatrix} [25.8,25.8] & [3.7,3.7] & [2.85,2.85] \\ [20,25] & [0.5,2] & [3,4] \\ [25,30] & [1,2] & [3,4] \\ [18,22] & [1,3] & [3,5] \\ [20,25] & [3,5] & [2,4] \end{bmatrix}$$

### 3.1 Standardization

Crack rate and IRI belongs to the Cost type indicator, standardizing with equation(2), SFC belongs to benefit index, standardizing with equation(1). So the normalized interval Numbers decision matrix B can be calculated:

$$B = \begin{bmatrix} [0.20,0.24] & [0.06,0.17] & [0.20,0.29] \\ [0.16,0.23] & [0.11,0.61] & [0.14,0.27] \\ [0.20,0.28] & [0.07,0.61] & [0.11,0.27] \\ [0.14,0.20] & [0.07,0.61] & [0.09,0.20] \\ [0.16,0.23] & [0.04,0.20] & [0.14,0.41] \end{bmatrix}$$

### 3.2 To determine the weights

Determined through the analytic hierarchy process (AHP), the attribute weights of each indicator is  $w_1=0.5$ ,  $w_2=0.2$ ,  $w_3=0.3$

So the decision-making evaluation matrix R is:

$$R = \begin{bmatrix} [0.100,0.120] & [0.012,0.034] & [0.060,0.087] \\ [0.080,0.115] & [0.024,0.122] & [0.042,0.081] \\ [0.100,0.140] & [0.014,0.122] & [0.033,0.081] \\ [0.070,0.100] & [0.014,0.122] & [0.027,0.060] \\ [0.080,0.115] & [0.008,0.040] & [0.042,0.123] \end{bmatrix}$$

### 3.3 build fuzzy soft set and make decisions

The five alternatives as filed, the three attributes as parameter set, basic fuzzy soft set (F, E) can be set up, such as table 3:

Table 3 basic fuzzy soft set (F, E)

U	$\epsilon_1$	$\epsilon_2$	$\epsilon_3$
$h_1$	[0.100,0.120]	[0.012,0.034]	[0.060,0.087]
$h_2$	[0.080,0.115]	[0.024,0.122]	[0.042,0.081]
$h_3$	[0.100,0.140]	[0.014,0.122]	[0.033,0.081]
$h_4$	[0.070,0.100]	[0.014,0.122]	[0.027,0.060]
$h_5$	[0.080,0.115]	[0.008,0.040]	[0.042,0.123]

In the table, filed is the five alternatives, namely  $U = \{h_1, h_2, h_3, h_4, h_5\}$ ; Parameter set E is the four decision attribute, namely  $E = \{\epsilon_1, \epsilon_2, \epsilon_3\}$ ,  $\epsilon_1$  said SFC,  $\epsilon_2$  said Crack rate,  $\epsilon_3$  said IRI.

So according to the data table 6, calculate by the equation (3), the choice value  $C_i$  of the five alternatives  $h_i (\forall h_i \in U)$  are:

$$C_1 = [0.172,0.241] \quad , \quad C_2 = [0.146,0.308] \quad , \\ C_3 = [0.147,0.343] \quad , \quad C_4 = [0.111,0.282] \quad , \\ C_5 = [0.130,0.278]$$

According to the choice value of the five roads, calculate by the formula (4), the decision values  $r_i$  of the 5 alternatives  $h_i (\forall h_i \in U)$  are:

$$r_1 = -0.093 \quad , \quad r_2 = 0.112 \quad , \quad r_3 = 0.292 \quad , \quad r_4 = -0.193 \quad , \\ r_5 = -0.118$$

According to the above decision value size:  $r_4 < r_5 < r_1 < r_2 < r_3$ , so the order of road maintenance plan is:  $h_3 \rightarrow h_2 \rightarrow h_1 \rightarrow h_5 \rightarrow h_4$ . That is to say, under the condition of the limited funding, road 3 should be maintained at the earliest.

## IV. Conclusions

An interval number multiple attribute decision making model has been established through applying with the analytic hierarchy process (AHP) and interval fuzzy soft set decision method. The model can determine the order of maintenance when decision makers only know the scope of the attribute value and do not know the specific value. This paper obtained the same sequence of maintenance with the literature

<sup>[8]</sup>, but the multiple attribute decision making method in the literature <sup>[8]</sup> need complex computation, this one is easier. The solution to the problem of multiple attribute decision making method to avoid the human subjectivity and randomness is put forward in this essay, and the results is more objective. The model provides a reference for the maintenance or rebuilding project decisions.

*university of technology*, 35 (3) ,2006, 108-111.

## References

- [1] LIU Changchun, QIN Renjie, 2014. (ed): *Pavement Maintenance Management and Maintenance Technology*, pp.155. People's Traffic Press, Beijing, China.
- [2] MU ChangChun, HE zhaoyi. A kind of new method of the road maintenance decision-making[J]. *Journal of Chongqing Jiaotong institute*, 19 (3) ,2000, 51-53.
- [3] HU Mingli, FAN Chengxian, SHI Kaiquan. The Property Analysis of the standardization method of the interval Numbers decision matrix [J]. *Journal of computer science*, 40 (10), 2013, 203-207.
- [4] LIAO Hongqiang, QIU Yong, YANG Xia, WANG Xinggang, GE Renwei. Study on determining the weight for applicating of analytic hierarchy process (AHP)[J]. *Mechanical engineers*, 6,2012,22-25.
- [5] Molodtsov D, Softset Theory-first Results[J]. *Computers and Mathematics with Applications*, 37(4), 1999, 19-31
- [6] P.K. Maji, R. Biswas, A.R. Roy. Fuzzy Soft Sets[J]. *The Journal of Fuzzy Mathematics*, 9(3), 2001, 589-602
- [7] YANG Xibei, Tsau YOUNG Lin, YANG Jingyu, LI Yan , YU Dongjun. Combination of interval-valued fuzzy set and soft set[J]. *Computers and Mathematics with Applications* , 58,2009, 521-527.
- [8] WANG Chao-hui, WANG Xuan-cang, MA Shi-bin, YUAN Yu-qing. The decision optimization of roads maintenance with uncertain property[J]. *Journal of Hebei*